

Massive particle production to NNLO in QCD

G. Chachamis^a *

^a Institut für Theoretische Physik und Astrophysik, Universität Würzburg
Am Hubland, D-97074 Würzburg, Germany

We discuss the recent derivation of the one-loop squared virtual QCD corrections to the W boson pair production in the quark-anti-quark-annihilation channel in the limit where all kinematical invariants are large compared to the mass of the W boson. In particular, we elaborate on the combined use of the helicity matrix formalism with the Mellin-Barnes representations technique.

1. Introduction

The Large Hadron Collider (LHC) is expected to have a huge impact on particle physics phenomenology. Most of the processes which will be studied at the LHC need to be calculated at least to next-to-leading order (NLO) in QCD whereas there are some for which a theoretical prediction is needed to next-next-to-leading order (NNLO). Electroweak gauge boson pair production falls into the latter category. One of the reasons is that the increase of the centre-of-mass energy at the LHC with respect to the Tevatron from 1.96 TeV to 14 TeV will result in a huge boost of the available data.

The importance of hadronic W-pair production is two-fold. Firstly, it is a process which allows the measurement of the vector boson trilinear couplings and therefore a comparison with the Standard Model (SM) predictions. Most attempts to model New Physics, such as Supersymmetry and Extra-dimensions in all variations, should be able to explain any deviations by consistently adjusting the anomalous couplings and/or by incorporating decays of new particles into vector boson pairs [1, 2].

Secondly, hadronic W pair production is important for investigations of the nature of the Electroweak symmetry mechanism by contributing the dominant background for the Higgs boson

mediated process (see Refs. [3–15]),

$$pp \rightarrow H \rightarrow W^*W^* \rightarrow l\bar{\nu}l'\nu',$$

in the Higgs mass range between $140 \text{ GeV} < M_H < 180 \text{ GeV}$ [16].

The interest in hadronic W pair production is well displayed by the fact that the Born cross section was calculated some thirty years ago [17]. The NLO QCD corrections were computed in the 90's and seen to contribute a 30% [18–22]. Next, soft gluon resummation effects were considered in Ref. [23] whereas massless fermion-boson scattering was studied at NNLO in Ref. [24]. The first steps towards a complete NNLO study were taken with the computation of the NNLO two-loop [25–27], as well as the one-loop squared [28] virtual corrections in a high energy expansion, $M_W^2 \ll s, t, u$.

The methods used in [28], differ somehow from the ones employed in [26] though, both are a continuation of the techniques used before in [29–33]. The difference lies mainly in the fact that for the one-loop squared corrections the helicity matrix formalism was additionally used to reduce the problem to a small set of integrals, which in turn were treated with Mellin-Barnes (MB) representations [34, 35]. The latter were constructed by means of the **MBrepresentation** package [36] and then analytically continued in the number of space-time dimensions $D = 4 - 2\epsilon$ using the **MB** package [37]. After the asymptotic expansion in the mass parameter, contours were closed and integrals finally resummed either with the help of

*This work was supported by the Sofja Kovalevskaja Award of the Alexander von Humboldt Foundation.

XSummer [38] or the **PSLQ** algorithm [39].

Here, we are going to give more details on how the two methods of the helicity formalism and MB representations were combined for the derivation of the result in [28]. We provide, as an example, the coefficient of a certain helicity matrix element for the one-loop amplitude in the high energy limit. This result is in closed analytic form expressed through harmonic polylogarithms and transcendental constants.

2. The Calculation

We shall introduce here part of the notation used in [28]. The charged vector-boson production in the leading partonic scattering process corresponds to

$$q_j(p_1) + \bar{q}_j(p_2) \rightarrow W^-(p_3, m) + W^+(p_4, m), \quad (1)$$

where p_i denote the quark and W momenta, m is the mass of the W boson and j is a flavour index. Here we are considering down-type quark scattering. Energy-momentum conservation implies

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu. \quad (2)$$

We consider the scattering amplitude \mathcal{M} for the process (1) at fixed values of the external parton momenta p_i , thus $p_1^2 = p_2^2 = 0$ and $p_3^2 = p_4^2 = m^2$. The amplitude \mathcal{M} may be written as a series expansion in the strong coupling α_s ,

$$|\mathcal{M}\rangle = \left[|\mathcal{M}^{(0)}\rangle + \left(\frac{\alpha_s}{2\pi} \right) |\mathcal{M}^{(1)}\rangle + \left(\frac{\alpha_s}{2\pi} \right)^2 |\mathcal{M}^{(2)}\rangle + \mathcal{O}(\alpha_s^3) \right]. \quad (3)$$

For convenience, we define the function $\mathcal{A}(\epsilon, m, s, t, \mu)$ for the squared amplitudes summed over spins and colors as

$$\sum_{\text{spin, color}} |\mathcal{M}(q_j + \bar{q}_j \rightarrow W^+ + W^-)|^2 = \mathcal{A}(\epsilon, m, s, t, \mu). \quad (4)$$

\mathcal{A} is a function of the Mandelstam variables s , t and u given by $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2 - m^2$ and $u = (p_1 - p_4)^2 - m^2$ and has a perturbative

expansion similar to Eq. (3)

$$\mathcal{A}(\epsilon, m, s, t, \mu) = \left[\mathcal{A}^{(0)} + \left(\frac{\alpha_s}{2\pi} \right) \mathcal{A}^{(1)} + \left(\frac{\alpha_s}{2\pi} \right)^2 \mathcal{A}^{(2)} + \mathcal{O}(\alpha_s^3) \right]. \quad (5)$$

In terms of the amplitudes the expansion coefficients in Eq. (5) may be expressed as

$$\mathcal{A}^{(0)} = \langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle, \quad (6)$$

$$\mathcal{A}^{(1)} = \left(\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(1)} | \mathcal{M}^{(0)} \rangle \right), \quad (7)$$

$$\mathcal{A}^{(2)} = \left(\langle \mathcal{M}^{(1)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle + \langle \mathcal{M}^{(2)} | \mathcal{M}^{(0)} \rangle \right). \quad (8)$$

As already mentioned, in order to compute $\langle \mathcal{M}^{(1)} | \mathcal{M}^{(1)} \rangle$ we used the helicity matrix formalism, namely we expressed the result in terms of helicity amplitudes, $\mathcal{M}^g(\lambda_1, \lambda_2, s, t)$. The quark and the anti-quark have opposite helicities in the centre-of-mass system so one helicity label above, $g = \pm 1$, suffices. λ_1 and λ_2 stand for the helicities of the W^+ and W^- respectively.

Starting from the one-loop amplitude, the initial expression can be rearranged as

$$|\mathcal{M}^{(1)}\rangle = \sum_{i,g} C_i^g(s, t, u, m) \mathcal{M}_i^g, \quad (9)$$

where the C_i are coefficients and \mathcal{M}_i^g are helicity matrix elements and $g = \pm$. The ten helicity matrix elements have been taken as defined in Ref. [40] (see also [41]):

$$\begin{aligned} \mathcal{M}_0^g &= \bar{v}(p_2) \not{\epsilon}_1 (\not{p}_3 - \not{p}_2) \not{\epsilon}_2 \mathcal{P}_g u(p_1), \\ \mathcal{M}_1^g &= \bar{v}(p_2) \not{p}_3 \mathcal{P}_g u(p_1) \epsilon_1 \cdot \epsilon_2, \\ \mathcal{M}_2^g &= \bar{v}(p_2) \not{\epsilon}_1 \mathcal{P}_g u(p_1) \epsilon_2 \cdot p_3, \\ \mathcal{M}_3^g &= -\bar{v}(p_2) \not{\epsilon}_2 \mathcal{P}_g u(p_1) \epsilon_1 \cdot p_4, \\ \mathcal{M}_4^g &= \bar{v}(p_2) \not{\epsilon}_1 \mathcal{P}_g u(p_1) \epsilon_2 \cdot p_1, \\ \mathcal{M}_5^g &= -\bar{v}(p_2) \not{\epsilon}_2 \mathcal{P}_g u(p_1) \epsilon_1 \cdot p_2, \\ \mathcal{M}_6^g &= \bar{v}(p_2) \not{p}_3 \mathcal{P}_g u(p_1) \epsilon_1 \cdot p_2 \epsilon_2 \cdot p_1, \\ \mathcal{M}_7^g &= \bar{v}(p_2) \not{p}_3 \mathcal{P}_g u(p_1) \epsilon_1 \cdot p_2 \epsilon_2 \cdot p_3, \\ \mathcal{M}_8^g &= \bar{v}(p_2) \not{p}_3 \mathcal{P}_g u(p_1) \epsilon_1 \cdot p_4 \epsilon_2 \cdot p_1, \\ \mathcal{M}_9^g &= \bar{v}(p_2) \not{p}_3 \mathcal{P}_g u(p_1) \epsilon_1 \cdot p_4 \epsilon_2 \cdot p_3, \end{aligned} \quad (10)$$

where $\mathcal{P}_g = \mathcal{P}_\pm = \frac{1 \pm \gamma_5}{2}$. All colour indices as well as the arguments of the polarization vectors, $\epsilon_1(p_3, \lambda_1)$ and $\epsilon_2(p_4, \lambda_2)$, have been suppressed.

Let us have a look at the $C_0^-(s, t, u, m)$ coefficient of the \mathcal{M}_0^- matrix element for the one-loop amplitude. Typically, $C_0^-(s, t, u, m)$ is given by

$$C_0^-(s, t, u, m) = \sum_i c_{0,i}(s, t, u, m) \mathcal{I}_i(s, t, u, m; \mu^2) \quad (11)$$

where $c_{0,i}(s, t, u, m)$ are polynomials in the kinematical variables and $\mathcal{I}_i(s, t, u, m; \mu^2)$ are one-loop integrals. For example, one of the one-loop integrals appearing in Eq. 11 is

$$I = \int d^d k \frac{k \cdot p_3}{k^2(k+p_1)^2(k+p_4)^2} \quad (12)$$

After feeding this into the **MBrepresentation** package, one gets, from the reduction of the original tensor structure into scalar objects, the following two terms:

$$\begin{aligned} I_1^{\text{MB}} &= \int_{a-i\infty}^{a+i\infty} dz_1 (-m^2)^{z_1} (-u)^{-1-\epsilon-z_1} \\ &\quad \times \Gamma(1-\epsilon) \Gamma(-\epsilon-z_1) \Gamma(-z_1) \\ &\quad \times \Gamma(1+z_1) \Gamma(1+\epsilon+z_1) \\ &\quad \times (\Gamma(2-2\epsilon))^{-1} \end{aligned} \quad (13)$$

and

$$\begin{aligned} I_2^{\text{MB}} &= \int_{b-i\infty}^{b+i\infty} dz_1 (-m^2)^{z_1} (-u)^{-1-\epsilon-z_1} \\ &\quad \times \Gamma(-\epsilon) \Gamma(1-\epsilon-z_1) \Gamma(-z_1) \\ &\quad \times \Gamma(1+z_1) \Gamma(1+\epsilon+z_1) \\ &\quad \times (\Gamma(2-2\epsilon))^{-1}, \end{aligned} \quad (14)$$

where of course $I = I_1^{\text{MB}} + I_2^{\text{MB}}$. One then needs to perform an asymptotic expansion in the mass parameter and finally resum the MB integrals. This allows to compute the coefficients C_i^g in a closed analytic form in the high energy limit. As an example, we present here the result up to $\mathcal{O}(\epsilon^2)$ for C_0^- (C_F the color factor and g_{WL}^2 the quark-W coupling)

$$\begin{aligned} C_0^- &= C_F g_{\text{WL}}^2 \left\{ \frac{1}{\epsilon^2} \left[-\frac{1}{1-x} \right] + \frac{1}{\epsilon} \left[-\frac{3}{2(1-x)} \right] \right. \\ &\quad \left. + \frac{i\pi}{\epsilon} \left[-\frac{1}{1-x} \right] + \left[\frac{7\pi^2}{12(1-x)} \right] \right\} \end{aligned}$$

$$\begin{aligned} &+ \frac{L_y^2}{2(1-x)} + \frac{3L_y}{2(1-x)} - \frac{9}{2(1-x)} \Big] \\ &+ i\pi \left[\frac{L_y}{1-x} \right] + \epsilon \left[-\frac{L_y^3}{3(1-x)} \right. \\ &+ \frac{1}{4} \left(-\frac{10}{x} - \frac{3}{1-x} \right) L_y^2 + \left(\frac{9}{2(1-x)} \right. \\ &- \frac{\pi^2}{2(1-x)} \Big) L_y + \frac{\pi^2}{8(1-x)} + \frac{1}{3} \left(\frac{7\zeta_3}{1-x} \right. \\ &- \frac{27}{1-x} \Big) + \frac{S_{1,2}(x)}{1-x} \Big] + i\pi \epsilon \left[\frac{\pi^2}{4(1-x)} \right. \\ &- \frac{L_y^2}{2(1-x)} - \frac{\text{Li}_2(x)}{1-x} - \frac{5L_y}{x} \Big] \\ &+ \epsilon^2 \left[\frac{3\pi^2}{8(1-x)} + \frac{\text{Li}_2(x)\pi^2}{2(1-x)} \right. \\ &- \frac{73\pi^4}{1440(1-x)} + \frac{L_y^4}{8(1-x)} \\ &+ \frac{1}{12} \left(\frac{20}{x} + \frac{3}{1-x} \right) L_y^3 \\ &+ \left(\frac{5\pi^2}{24(1-x)} - \frac{3}{4} \left(\frac{8}{x} + \frac{3}{1-x} \right) \right) L_y^2 \\ &+ \frac{1}{2} \left(\frac{7\zeta_3}{1-x} - \frac{36}{1-x} \right) + \left(\frac{1}{8} \left(\frac{20}{x} \right. \right. \\ &- \frac{1}{1-x} \Big) \pi^2 + \frac{9}{1-x} \Big) L_y - \frac{L_y S_{1,2}(x)}{1-x} \\ &- \frac{5S_{1,2}(x)}{x} - \frac{S_{1,3}(x)}{1-x} + \frac{S_{2,2}(x)}{1-x} \Big] \\ &+ i\pi \epsilon^2 \left[\frac{L_y^3}{6(1-x)} + \frac{5L_y^2}{2x} \right. \\ &+ \left(-\frac{\pi^2}{4(1-x)} - \frac{12}{x} \right) L_y + \frac{\text{Li}_2(x)L_y}{1-x} \\ &+ \frac{7\zeta_3}{3(1-x)} + \frac{5\text{Li}_2(x)}{x} \\ &- \frac{\text{Li}_3(x)}{1-x} + \frac{S_{1,2}(x)}{1-x} \Big] \Big\}, \end{aligned} \quad (15)$$

where we have defined

$$x = -\frac{t}{s}, \quad y = -\frac{u}{s}, \quad m_s = \frac{m^2}{s} \quad (16)$$

and

$$L_m = \text{Log}(m_s), \quad L_y = \text{Log}(1-x). \quad (17)$$

Similarly, one can compute the C_i^g coefficients for all the helicity matrix elements \mathcal{M}_i^g . The complex parts of the coefficients are given explicitly as can be seen in Eq. 15 which means that obtaining the complex conjugate expressions for the tree-level, $|\mathcal{M}^{(0)}\rangle$, and the one loop, $|\mathcal{M}^{(1)}\rangle$, amplitudes is trivial. The following step would be the contraction $\langle\mathcal{M}^{(1)}|\mathcal{M}^{(1)}\rangle$. As an easy test, we have checked $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$ derived with this method against the results provided in [18,19].

In order to check $\langle\mathcal{M}^{(1)}|\mathcal{M}^{(1)}\rangle$ we have used a more involved test of the infrared structure according to the Catani prediction [42]. In the case of one-loop QCD amplitudes, their poles in ϵ can be expressed as a universal combination of the tree amplitude and a colour-charge operator $\mathbf{I}^{(1)}(\epsilon)$. The generic form of $\mathbf{I}^{(1)}(\epsilon)$ was found by Catani and Seymour [43].

3. Conclusions

We have discussed some details of the computation of the one-loop squared NNLO QCD virtual corrections for the process $q\bar{q} \rightarrow W^+W^-$ in the limit of small vector boson mass. Our main result was presented in [28]. This was a second step towards the complete evaluation of the virtual corrections. In a forthcoming publication, we will derive a series expansion in the mass and integrate the result numerically to recover the full mass dependence, similarly to what has been done in [44].

To complete the NNLO project one still needs to consider $2 \rightarrow 3$ real-virtual contributions and $2 \rightarrow 4$ real ones. The real-virtual corrections are known from the NLO studies on $WW + jet$ production in [45,46].

REFERENCES

1. CDF Collaboration, Phys. Rev. Lett. **94** (2005) 211801
2. D0 Collaboration, Phys. Rev. Lett. **94** (2005) 151801
3. M. Spira, A. Djouadi, D. Graudenz and P. M. Zerwas, Nucl. Phys. B **453** (1995) 17
4. S. Dawson, Nucl. Phys. B **359** (1991) 283
5. R. V. Harlander and W. B. Kilgore, Phys. Rev. Lett. **88** (2002) 201801
6. C. Anastasiou and K. Melnikov, Nucl. Phys. B **646** (2002) 220
7. V. Ravindran, J. Smith and W. L. van Neerven, Nucl. Phys. B **665** (2003) 325
8. S. Catani, D. de Florian and M. Grazzini, JHEP **0201** (2002) 015
9. G. Davatz, G. Dissertori, M. Dittmar, M. Grazzini and F. Pauss, JHEP **0405** (2004) 009
10. C. Anastasiou, K. Melnikov and F. Petriello, Phys. Rev. Lett. **93** (2004) 262002
11. C. Anastasiou, G. Dissertori and F. Stockli, arXiv:0707.2373 [hep-ph]
12. M. Grazzini, arXiv: 0801.3232 [hep-ph]
13. A. Bredenstein, A. Denner, S. Dittmaier and M. M. Weber, Phys. Rev. D **74** (2006) 013004
14. T. Binoth, M. Ciccolini, N. Kauer and M. Kramer, JHEP **0503** (2005) 065 [arXiv:hep-ph/0503094]
15. T. Binoth, M. Ciccolini, N. Kauer and M. Kramer, JHEP **0612** (2006) 046 [arXiv:hep-ph/0611170]
16. M. Dittmar and H. K. Dreiner, Phys. Rev. D **55** (1997) 167 [arXiv:hep-ph/9608317]
17. R. W. Brown and K. O. Mikaelian, Phys. Rev. D **19** (1979) 922
18. J. Ohnemus, Phys. Rev. D **44** (1991) 1403
19. S. Frixione, Nucl. Phys. B **410** (1993) 280
20. L. J. Dixon, Z. Kunszt and A. Signer, Nucl. Phys. B **531** (1998) 3 [arXiv:hep-ph/9803250]
21. L. J. Dixon, Z. Kunszt and A. Signer, Phys. Rev. D **60** (1999) 114037 [arXiv:hep-ph/9907305]
22. J. M. Campbell and R. K. Ellis, Phys. Rev. D **60** (1999) 113006 [arXiv:hep-ph/9905386]
23. M. Grazzini, JHEP **0601** (2006) 095
24. C. Anastasiou, E. W. N. Glover and M. E. Tejeda-Yeomans, Nucl. Phys. B **629** (2002) 255
25. G. Chachamis, Acta Phys. Polon. B **38** (2007) 3563 arXiv:0710.3035 [hep-ph]
26. G. Chachamis, M. Czakon and D. Eiras, arXiv:0802.4028 [hep-ph]
27. G. Chachamis, arXiv:0807.0548 [hep-ph]
28. G. Chachamis, M. Czakon and D. Eiras, arXiv:0806.3043 [hep-ph]
29. M. Czakon, A. Mitov and S. Moch, Phys. Lett. B **651** (2007) 147, arXiv:0705.1975 [hep-ph]

- ph]
30. M. Czakon, A. Mitov and S. Moch, arXiv:0707.4139 [hep-ph]
 31. M. Czakon, J. Gluza and T. Riemann, Phys. Rev. D **71** (2005) 073009, hep-ph/0412164
 32. M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. B **751** (2006) 1, hep-ph/0604101
 33. S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. B **786** (2007) 26
 34. V.A. Smirnov, Phys. Lett. B **460** (1999) 397, hep-ph/9905323
 35. J.B. Tausk, Phys. Lett. B **469** (1999) 225, hep-ph/9909506
 36. G. Chachamis and M. Czakon, `MBrepresentation.m`, Unpublished
 37. M. Czakon, Comput. Phys. Commun. **175** (2006) 559, hep-ph/0511200
 38. S. Moch and P. Uwer, Comput. Phys. Commun. **174** (2006) 759, math-ph/0508008
 39. H.R.P. Ferguson and D.H. Bailey, (1992), (see e.g. <http://mathworld.wolfram.com/PSLQAlgorithm.html>)
 40. K. P. O. Diener, B. A. Kniehl and A. Pilaftsis, Phys. Rev. D **57** (1998) 2771
 41. A. Denner and T. Sack, Nucl. Phys. B **306**, 221 (1988)
 42. S. Catani, Phys. Lett. B **427** (1998) 161 [arXiv:hep-ph/9802439].
 43. S. Catani and M. H. Seymour, Nucl. Phys. B **485** (1997) 291 [Erratum-ibid. B **510** (1998) 503]
 44. M. Czakon, arXiv:0803.1400 [hep-ph]
 45. J. M. Campbell, R. K. Ellis and G. Zanderighi, JHEP **0712** (2007) 056
 46. S. Dittmaier, S. Kallweit and P. Uwer, Phys. Rev. Lett. **100** (2008) 062003